JOURNAL OF MATERIALS SCIENCE 26 (1991) 253–257

Subcritical crack growth and threshold in borosilicate glass

T. FETT, K. GERMERDONK, A. GROSSMÜLLER, K. KELLER, D. MUNZ
Kernforschungszentrum Karlsruhe, Institut für Material- und Festkörperforschung IV, Universität Karlsruhe, Institut für Zuverlässigkeit und Schadenskunde im Maschinenbau, Postfach 3640, D-7500 Karlsruhe, FRG

The crack-growth behaviour of borosilicate glass was investigated by use of lifetime measurements on Knoop-damaged specimens. The threshold behaviour in particular was observed, and the influence of residual wedging stresses and preloading below the threshold value were studied in detail. A threshold of $K_{\mathrm{Ith}} = 0.38$ MPa m$^{1/2}$ was found. The residual stress intensity factor caused by the Knoop indentations was found to be $K_{\mathrm{Ires,0}} \simeq 0.1$ MPa m$^{1/2}$. In a static test performed below the threshold, the threshold value was increased by $\Delta K_{\mathrm{Ith}} \simeq 0.15$ MPa m$^{1/2}$.

1. Introduction

Glass exhibits pronounced subcritical crack growth. The knowledge of this crack-growth behaviour is of great interest, especially in the range of extremely low velocities. Low crack-growth rates are necessary for lifetime predictions of components. Therefore, measurements of subcritical crack growth must cover the range of lowest possible stress intensity factors. At low stress intensity factors, glass shows a characteristic threshold behaviour. The knowledge of the threshold in combination with a proof-test allows the design of components taking into account K_{Ith} instead of K_{Ic} in order to avoid any delayed failure.

As shown by Wiederhorn and Bolz [1], double cantilever beam measurements on soda-lime glass exhibit a threshold of $K_{\mathrm{Ith}} = 0.25$ MPa m$^{1/2}$. From lifetime measurements carried out on borosilicate glass, a threshold of $K_{\mathrm{Ith}} = 0.38$ MPa m$^{1/2}$ was found [2].

2. Procedure for determining the v–K curve

2.1. General relations

A lifetime method was developed, to determine extremely low crack growth rates down to 10^{-12} to 10^{-13} m sec^{-1} [2, 3]. The procedure is based on the general lifetime equation for tests under static load, σ

$$t_{\mathrm{f}} = \frac{2}{\sigma^2 Y_{\mathrm{i}}^2} \int_{K_{\mathrm{Ii}}}^{K_{\mathrm{Ic}}} \frac{K_{\mathrm{I}}}{v(K_{\mathrm{I}})} \, \mathrm{d}K_{\mathrm{I}} \qquad (1)$$

where t_{f} is the lifetime, K_{Ii} the stress intensity factor, and Y_{i} the related geometric function at the moment of load application. Because the stress intensity factor is given by

$$K_{\mathrm{I}} = \sigma Y a^{1/2} \qquad (2)$$

it holds for the initial value

$$K_{\mathrm{Ii}} = \sigma Y_{\mathrm{i}} a_{\mathrm{i}}^{1/2} \qquad (3)$$

with the initial crack depth a_{i}. For semi-elliptical surface cracks the function Y depends on the relative crack depth a/t and the aspect ratio a/c, where t is the thickness of a specimen and $2c$ is the width of the crack, i.e. $Y = Y(a/c, a/t)$.

The quantity

$$t_{\mathrm{f}} \sigma^2 Y_{\mathrm{i}}^2 = 2 \int_{K_{\mathrm{Ii}}}^{K_{\mathrm{Ic}}} \frac{K_{\mathrm{I}}}{v(K_{\mathrm{I}})} \, \mathrm{d}K_{\mathrm{I}} \qquad (4)$$

is only a function of the initial stress intensity factor, K_{Ii}. Therefore, it follows for the initial crack growth rate

$$v(K_{\mathrm{Ii}}) = -2 \frac{a_{\mathrm{i}}}{t_{\mathrm{f}}} \frac{\mathrm{d}[\log K_{\mathrm{Ii}}]}{\mathrm{d}[\log(\sigma^2 t_{\mathrm{f}} Y_{\mathrm{i}}^2)]} \qquad (5)$$

When the lifetime is measured in a static test with the stress, σ, and the initial crack geometry $(a_{\mathrm{i}}, c_{\mathrm{i}})$ is determined on the fracture surface, K_{Ii} is given by Equation 3 and all quantities occurring in Equation 5 are known. A procedure of evaluating Equation 5 is given in detail elsewhere [2, 3].

2.2. Geometric function

The solution proposed by Newman and Raju [4–6] is applied most frequently to calculate local stress intensity factors of semi-elliptical surface cracks. Alternatively to the local stress intensity factors, the application of so-called "averaged stress intensity factors" is usual in evaluating two-dimensional crack-growth problems. One possibility is to define averaged stress intensity factors by the energy release rate [7], i.e.

$$K_{\mathrm{A}}^* = \left(\frac{4}{\pi} \int_0^{\pi/2} K_{\mathrm{I}}^2 \sin^2\varphi \, \mathrm{d}\varphi \right)^{1/2} \qquad (6)$$

for the value at the deepest point of the semi-ellipse, and

$$K_{\mathrm{B}}^* = \left(\frac{4}{\pi} \int_0^{\pi/2} K_{\mathrm{I}}^2 \cos^2\varphi \, \mathrm{d}\varphi \right)^{1/2} \qquad (7)$$

TABLE I Coefficients of Equation 8 for point A

μ	$v=0$	$v=1$	$v=2$	$v=3$
0	1.9807	−2.3985	4.1946	−4.0814
1	−1.1607	1.5134	−8.0706	7.5993
2	0.4119	−0.5798	5.3230	−4.7820
3	−0.0570	0.0700	−1.1790	1.0455

TABLE II Coefficients of Equation 8 for point B

μ	$v=0$	$v=1$	$v=2$	$v=3$
0	1.3669	−1.5697	3.5106	−2.6571
1	−0.0471	1.0446	−5.7531	4.7531
2	−0.1473	−0.5130	3.5505	−3.0005
3	0.0443	0.0924	−0.7543	0.6617

for the surface points. The geometric functions calculated by Equations 6 and 7 can be expressed for $0.3 < a/c < 2$ and $a/t < 0.25$ within $\pm 0.5\%$ deviations by the relation

$$Y^*_{\mathrm{A,B}} = \sum_{v,\mu=0}^{3} C_{v,\mu} \left(\frac{a}{c}\right)^{\mu} \left(\frac{a}{t}\right)^{v} \tag{8}$$

with the coefficients listed in Tables I and II.

3. *v–K* curve for borosilicate glass

Lifetime measurements were carried out with 4 mm × 5 mm × 45 mm specimens of three heats (1, 2 and 3) of a commercial borosilicate glass (BK 7, Glaswerke Schott, Mainz) with a transition temperature of 560 °C. The polished specimens were annealed at 450 °C for 5 h and then damaged by Knoop indentation with an indentation load of 50 N. To remove the residual stresses remaining after indentation, the specimens were again annealed at 450 °C. Then static four-point-bending tests were performed with the indentation cracks in the tensile region, and the lifetimes were determined. Using the procedure described in [2] the crack growth rates were determined. To evaluate Equation 5 the logarithmic derivative $\mathrm{d}(\log K_{\mathrm{Ii}})/\mathrm{d}(\log(\sigma^2 t_{\mathrm{f}} Y_{\mathrm{i}}^2))$ has to be determined. This can be done by plotting the lifetime quantity $t_{\mathrm{f}}\sigma^2 Y_{\mathrm{i}}^2$ against the stress intensity factor K_{Ii}. The result obtained for heat 3 is shown in Fig. 1. The derivative can be calculated from the mean curve.

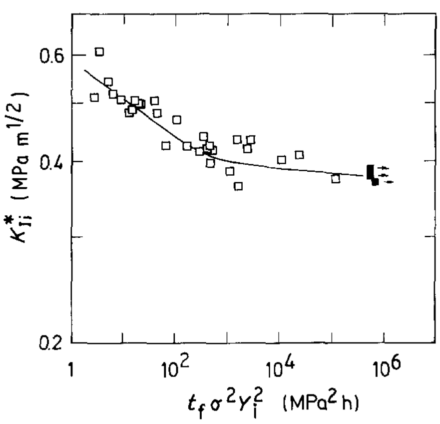

Figure 1 Lifetime quantity $t_{\mathrm{f}}\sigma^2 Y_{\mathrm{i}}^2$ plotted against stress intensity factor applied for heat 3.

The results obtained for the three different heats of glass are plotted in Fig. 2. Apart from some scatter, the behaviour is the same for all heats. There is an obvious threshold $K_{\mathrm{Ith}} \simeq 0.38$ MPa m$^{1/2}$. The complete v–K curve is given by

$$v = \begin{cases} AK_{\mathrm{I}}^{n} & K_{\mathrm{I}} > K_{\mathrm{Ith}} \\ 0 & K_{\mathrm{I}} < K_{\mathrm{Ith}} \end{cases} \tag{9}$$

with $A = 0.002\,65$ (MPa m sec) and $n \simeq 15$. The fracture toughness could be determined from the size and the aspect ratio of the semi-ellipse at the moment of failure. This final semi-ellipse is clearly visible as a sharp line on the fracture surface. It was found that $K_{\mathrm{Ic}} \simeq 0.8$ MPa m$^{1/2}$ for heat 1 [2], $K_{\mathrm{Ic}} \simeq 0.90$ MPa m$^{1/2}$ for heat 2, and $K_{\mathrm{Ic}} \simeq 1.05$ MPa m$^{1/2}$ for heat 3.

3.1. Influence of residual wedge stresses on the *v–K* curve

The influence of residual stresses caused by the indentation test on the v–K curve and threshold can be investigated by measuring the v–K curve for specimens where the wedging stresses have not been removed by annealing. The residual stresses caused by the strongly deformed region below the indentor (acting as a wedge) gives rise to a residual stress intensity factor $K_{\mathrm{Ires,0}}$. If the specimen is loaded by external stresses causing the stress intensity factor K_{Iappl}, the residual stresses become reduced until they disappear for $K_{\mathrm{Iappl}} = K_{\mathrm{Iopen}}$. Owing to the linear–elastic material behaviour in the area surrounding the wedge, the decrease becomes linear

$$K_{\mathrm{Ires}} = K_{\mathrm{Ires,0}}\left(1 - \frac{K_{\mathrm{Iappl}}}{K_{\mathrm{Iopen}}}\right) \tag{10}$$

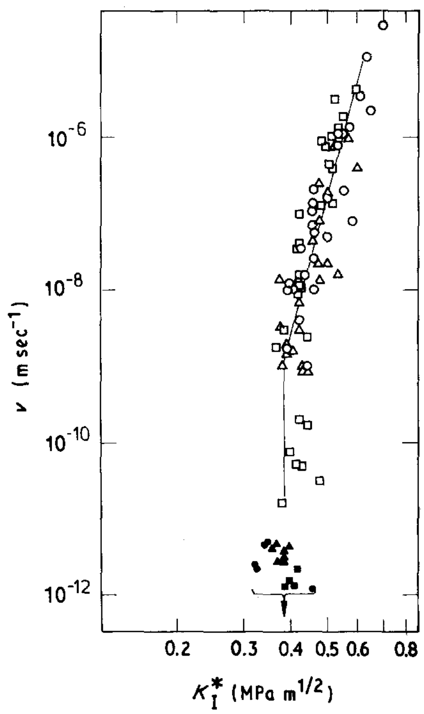

Figure 2 v–K curves of heats (○) 1 [2], (△) 2 and (□) 3 of borosilicate glass at 20 °C in deionized water.

The total stress intensity factor then results by superposition of applied and residual K

$$K_{\mathrm{I}} = \begin{cases} K_{\mathrm{Iappl}} + K_{\mathrm{Ires,0}}\left(1 - \dfrac{K_{\mathrm{Iappl}}}{K_{\mathrm{Iopen}}}\right) & \text{for } K_{\mathrm{Iappl}} < K_{\mathrm{Iopen}} \\ K_{\mathrm{Iappl}} & \text{for } K_{\mathrm{Iappl}} \geq K_{\mathrm{Iopen}} \end{cases} \tag{11}$$

This reduction of internal stress intensity factors by external stresses (a consequence of simple elasticity) is often completely ignored and K_{Ires} assumed as a constant.

The influence of this behaviour on the measured v–K curves is illustrated schematically in Fig. 3 where the subcritical crack growth rates are plotted against the stress intensity factor applied K_{Iappl}. The left-hand scheme shows the case $K_{\mathrm{Iopen}} > K_{\mathrm{Ic}}$, which means the crack will not open before and K_{Ires} will not vanish before the specimen fails. In that case, two separated v–K curves must be found for the specimens with internal stresses and the specimens without internal stresses.

In the right-hand scheme, the case $K_{\mathrm{Iopen}} < K_{\mathrm{Ic}}$ is represented. From the point where $K_{\mathrm{Iappl}} = K_{\mathrm{Iopen}}$ both curves must coincide.

In Fig. 4 measurements of crack growth rates are shown. The right-hand curve represents measurements obtained with annealed specimens (heat 2, Fig. 2). The data on the left were obtained with a second series of specimens, which were not annealed before the lifetime tests. Both curves are different over the whole range of measurements and show the behaviour characteristic of $K_{\mathrm{Iopen}} > K_{\mathrm{Ic}}$. The apparent threshold for the specimens with residual stresses is about $K_{\mathrm{Ith,res}} \simeq 0.28\ \mathrm{MPa\,m^{1/2}}$. The specimens without residual stresses exhibit a value of $K_{\mathrm{Ith}} = 0.38\ \mathrm{MPa\,m^{1/2}}$. Therefrom a residual stress intensity factor at $K_{\mathrm{Iappl}} = K_{\mathrm{Ith,res}}$ of $K_{\mathrm{Ires}} = 0.1\ \mathrm{MPa\,m^{1/2}}$ can be concluded. The residual stress intensity factor without an external load can be estimated by use of Equation 11. Because in the presented investigation $K_{\mathrm{Iopen}} > K_{\mathrm{Ic}}$, it results that

$$K_{\mathrm{Ires}} < K_{\mathrm{Ires,0}} < K_{\mathrm{Ires}} \Big/ \left(1 - \frac{K_{\mathrm{Ith,res}}}{K_{\mathrm{Ic}}}\right) \tag{12}$$

and thus

$$0.1\ \mathrm{MPa\,m^{1/2}} < K_{\mathrm{Ires,0}} < 0.145\ \mathrm{MPa\,m^{1/2}} \tag{13}$$

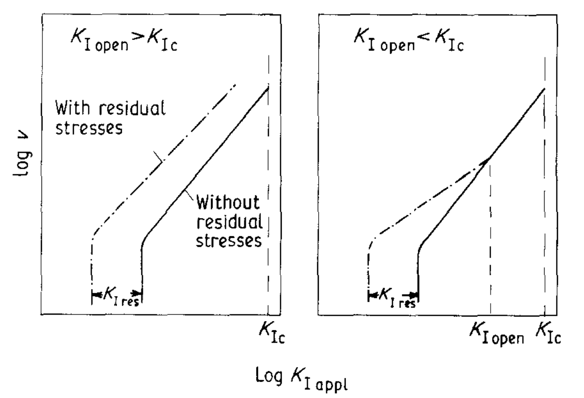

Figure 3 Influence of residual stresses on the v–K curve (schematic).

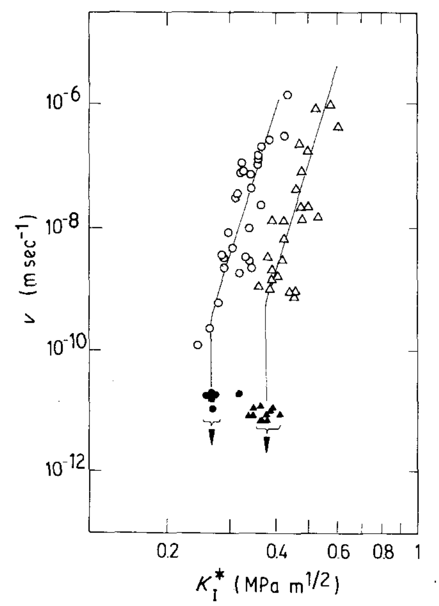

Figure 4 v–K curves for a series (○, ●) with and (△, ▲) without residual stresses.

This value is not necessarily a constant; it may depend on the type of glass as well as on the indentation load.

3.2. Influence of a long-term preloading with $K_{\mathrm{Iappl}} < K_{\mathrm{Ith}}$

If Knoop-damaged specimens are loaded with $K_{\mathrm{I}} > K_{\mathrm{Ith}}$ subcritical crack extension occurs. On the other hand, it is of interest whether a lower load $K_{\mathrm{I}} < K_{\mathrm{Ith}}$ will also affect the crack behaviour.

Two series of Knoop-cracked specimens were loaded in four-point bending tests in water. The first series was loaded normally and the lifetimes were measured (heat 3, Fig. 2). A second series was loaded for 400 h with a lower load so that $K_{\mathrm{I}} < K_{\mathrm{Ith}}$. After this preloading all specimens were loaded once more with higher loads and higher initial stress intensity factors K_{Ii}, and the lifetimes were determined.

Fig. 5 shows the normalized lifetimes $t_{\mathrm{f}}\sigma^2 Y^2$ as a function of the stress intensity factor applied in the

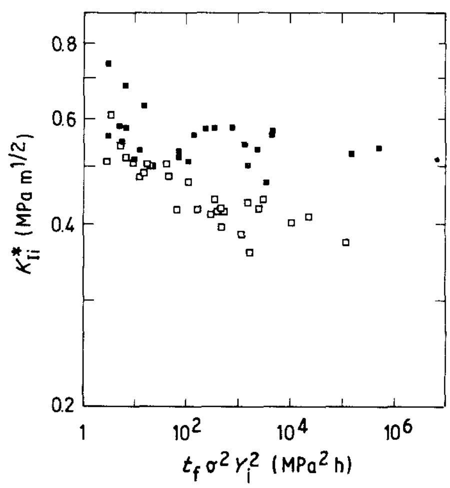

Figure 5 Normalized lifetimes for a series of (■) preloaded specimens compared with lifetimes of (□) normally prepared specimens.

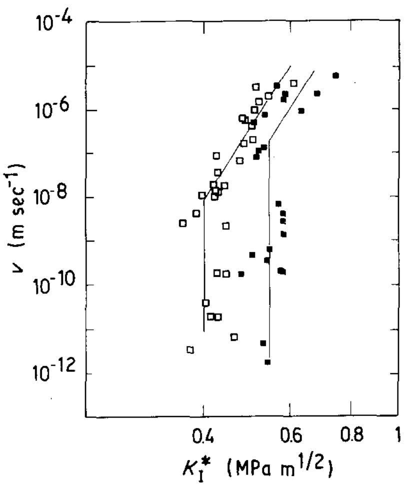

Figure 6 v–K curves of (■) preloaded and (□) normally prepared specimens.

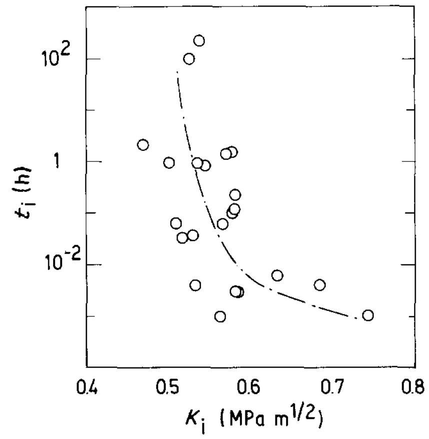

Figure 7 Crack initiation time as a function of nominal stress intensity factor applied in the second lifetime test of preloaded specimens.

preloaded series as well as in the series measured without preloading. It becomes quite evident that preloading increases the lifetimes or that an increasing stress intensity factor is necessary to obtain the same lifetime as found in the tests without preloading. This is "strengthening" of the preloaded specimens.

In Fig. 6 the resulting v–K curves are plotted for both series. Preloading causes a shift of the threshold by $\Delta K_{\text{Ith}} \simeq 0.15$ MPa m$^{1/2}$. The v–K curve at higher crack growth rate is nearly unaffected by preloading. According to theoretical considerations, both curves should become completely identical at high crack growth rates.

One reason for the shifted threshold might be the viscous nature of the glass. Because at $K_{\text{I}} < K_{\text{Ith}}$ no crack extension occurs there is enough time during preloading, and the high stresses at the crack tip can cause crack-tip blunting. In the next test the stress must be increased to a certain amount to create a new sharp crack tip. This increased stress results in an increased threshold value.

If this interpretation is true the treatment of the preloaded series is no longer a pure crack problem but at least partially a notch problem. The determination of the stress intensity factor requires the existence of a sharp crack. Here only a "nominal stress intensity factor" is obtained from Equation 2 for the case of the preloaded series, and only a nominal increase in the threshold value occurs. If, in addition, a crack initiation time, t_i, is necessary to create a sharp crack tip replacing the blunt crack, the crack growth rates also become wrong, because Equation 1 must now correctly read

$$t_{\text{f, preloaded}} = \frac{2}{\sigma^2 Y_i^2} \int_{K_{\text{Ii}}}^{K_{\text{Ic}}} \frac{K_{\text{I}}}{v(K_{\text{I}})} \, dK_{\text{I}} + t_i \quad (13)$$

The crack initiation time, t_i, may depend on the nominal K_{Ii} in the second lifetime test because a higher load will require a shorter time to create a sharp crack tip. Also, the stress intensity factor during the preload test and the duration of preloading can have an influence. A correct fracture-mechanical evaluation of Equation 13 presumes the knowledge of the dependency $t_i = f(K_{\text{Ii}})$.

Because the integral term in Equation 13, describing the "sharp crack behaviour", is known from tests without preloading as described by Equation 1, one obtains the crack initiation time from the data given in Fig. 5.

$$t_i(K_{\text{Ii}}) = t_f(K_{\text{Ii}})|_{\text{preloaded}} - t_f(K_{\text{Ii}}) \quad (14)$$

The result is given in Fig. 7 where the crack initiation time is plotted as a function of the nominal stress intensity factor applied in the lifetime test after preloading. There is an obvious trend of decreasing crack initiation time with increasing load. The scatter includes the influence of the individual preload stress intensity factor.

4. Conclusions

The subcritical crack growth and threshold behaviour of three heats of a borosilicate glass were investigated. The main results are as follows.

1. All heats showed a threshold value of $K_{\text{Ith}} \simeq 0.38$ MPa m$^{1/2}$.
2. The crack growth rates above K_{Ith} can be described by a power law (Equation 9) with $n = 15$ and $A = 0.00265$ (MPa m sec).
3. In specimens with residual stresses the wedging effects do not vanish before K_{Ic} is reached, i.e. $K_{\text{Iopen}} > K_{\text{Ic}}$.
4. In Knoop-damaged specimens where the residual stresses due to the indentation are not removed, the residual stress intensity factor is found to be

$$0.1 \text{ MPa m}^{1/2} < K_{\text{Ires},0} < 0.145 \text{ MPa m}^{1/2}$$

5. Preloading at stress intensity factors directly below the threshold causes a shift of the threshold value to higher nominal K values. The shift is found to be $\Delta K_{\text{Ith}} \simeq 0.15$ MPa m$^{1/2}$.

6. The influence of crack-tip blunting during the preloading time is discussed and a crack initiation time is observed.

References

1. S. M. WIEDERHORN and L. H. BOLZ, *J. Amer. Ceram. Soc.* **53** (1970) 543.
2. T. FETT, D. MUNZ and K. KELLER, *J. Mater. Sci.* **23** (1988) 798.
3. T. FETT and D. MUNZ, *Commun. Amer. Ceram. Soc.* **68** (1985) C213.
4. J. C. NEWMAN AND I. S. RAJU, NASA Technical Paper 1578 (1979).
5. *Idem, Engng Fract. Mech.* **15** (1981) 185.
6. T. FETT, *Int. J. Fract.* **33** (1987), R47–R50, corrigendum **35** (1987) R65.
7. T. A. CRUSE and P. M. BESUNER, *J. Aircraft* **12** (1975) 369.

Received 29 August 1989
and accepted 19 February 1990